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FUZZY EVALUATION USING VECTOR AND MATRIX VALUED MARKING METHOD

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ABSTRACT

Existing two systems of evaluation of interviewee written test for subjective questions in interview (grading method and traditional marking method) are considered, and a better method of evaluation called fem is suggested. Fem is a computer based fuzzy approach where a vector valued marking is used. Finally, fem is generalized and the generalized method is called gfem in which a matrix-valued marking is adopted.

Keywords: Fuzzy grading systems; Fuzzy reasoning; Fuzzy rules; Grade membership functions

1. INTRODUCTION

Fuzzy sets were initiated by Zadeh in 1965. Methods of approximate inference based on fuzzy set theory allow formal representation to be built for decision making procedures containing fuzzy premises. The vital aim of the fuzzy set theory would be, according to Zadeh, to symbolize how the human mind perceives and manipulate information. It is easy to understand that man commonly uses fuzzy concepts when he perceives the outside world and when he thinks. It seems that human brain processes many hedges like good, very good, similar to, tall, large, long, very tall, very long, almost similar, brilliant, extraordinary, more brilliant, etc. to name a few only out of infinite, more easily than numbers [11]. One of the most important facts of human thinking is its ability to review information (or, in the particular case here, it is evaluation) into levels of fuzzy sets which bear an approximate relation to the primary data. For more on fuzzy sets refer [2,3,9] and for fuzzy decision making refer [1,7,10]. Linguistic descriptions of complex situations or strategies generally include fuzzy concepts. To be able to represent and manipulate these we are to yield tools for modeling humanistic or man-machine systems in which information is often vague and thus respective evaluation is a challenge for usual techniques.

The idea of fuzzy sets and membership values provides a possible model for inexact concepts, that is it provide subjective judgements, for all types of evaluations. Our present day understanding of the nervous system suggests that the fuzzy paradigm would be appropriate to signify the knowledge based decision and to select proper evaluation method. Regrettably, the cerebral dynamic processes are of some complexity; and so exact decision procedure (from a random and heterogeneous mixture of hedges like good, very good, bad, very bad, satisfactory or unsatisfactory etc. to a numerical value) is practically impossible unless it is handled with fuzzy tools. Even then a lot of uncertainty is always present in the conclusion procedure but in a diluted form i.e., not in an exact form. We assert that a potentially fruitful explanation to the problem of finding out a better and better method of evaluation is nothing but a fuzzy approach. Because only it can deal with the vague information whereas crisp cannot. The pattern presented here is a

confluence of fuzzy set theory and it is potentially finer than both the existing method of evaluation: grading system and traditional marking system.

Consider the evaluation of interviewee'sanswer scripts by the two popular existing systems. One is the grading marking system and the other is the traditional marking system. Type of questions in the interview are assumed to be such that answers are of subjective types which may be short or long but not that of objective type (for example, putting a tick "√" mark or answering true or false and so on). The case, in which types of questions are such that candidate will be awarded either full mark or zero, are not considered here. That is because we are employing the concept of fuzzy. In traditional system of evaluation, "marking" i.e. awarding of marks (arithmetical) is done, whereas in the Grading system of evaluation, "grading" i.e. awarding of grades (equivalent to an interval of two integer numbers) is done. Grading system is introduced, in fact, due to interviewer's inability to measure with precision the human qualities with all their intricate levels of variation and mix up of observable as well as non-observable attributes. The details on evaluation for employees can be seen in [4,5,6,8]. We consider, for the purpose of an example only, the standard guidelines of a reputed company for evaluation of interviewee's performance, which are as furnished below in brief:

(i) Each question in the answer scripts will be awarded letter grade according to the level of performance judged by the interviewers/evaluators. (ii) For letter grade, a 5-point scale has been adopted. "A" being the highest and "E" being the lowest. They are assigning notional values.

(iii) Each answer should be checked thoroughly into its minute details and its strength and weaknesses may be marked to facilitate its overview. The level of performance may then be judged in terms of the hedges good, very good, excellent etc.

(iv) After deciding the level of performance in its qualitative terms, then the corresponding grade may be indicated against each answer to a question. Every answer to each question therefore, will have separate grades such as C, A, E, B, etc.

(v) These grades have to be combined into an Overall Grade for the whole answer script.

This grading system of evaluation in the interview involves many hedges or factors like good, bad, satisfactory, etc. which are the sources of vagueness and thus involve a substantial amount of fuzziness.

The fuzzy pattern of assessment presented here is abbreviated as fem, which stands for fuzzy evaluation method.The sequence of development of the methods of evaluation is shown in Fig. 1. In this fem, a vector approach for the representation of detailed interview performance is used. Lastly, fem is generalized by introducing the idea of a matrix-approach of marking, and the generalized fem is called generalized fuzzy evaluation method or in short gfem.

2. PRELIMINARIES

Here, in this section we shall introduce the basic concepts of fuzzy set, which will be used in the later section.

Definition 2.1.*Fuzzy set*: A fuzzy set A in a universe of discourse X is defined as the following set of pairs

$A = \{(\ _A(x),x):x \in X\}$, (1)

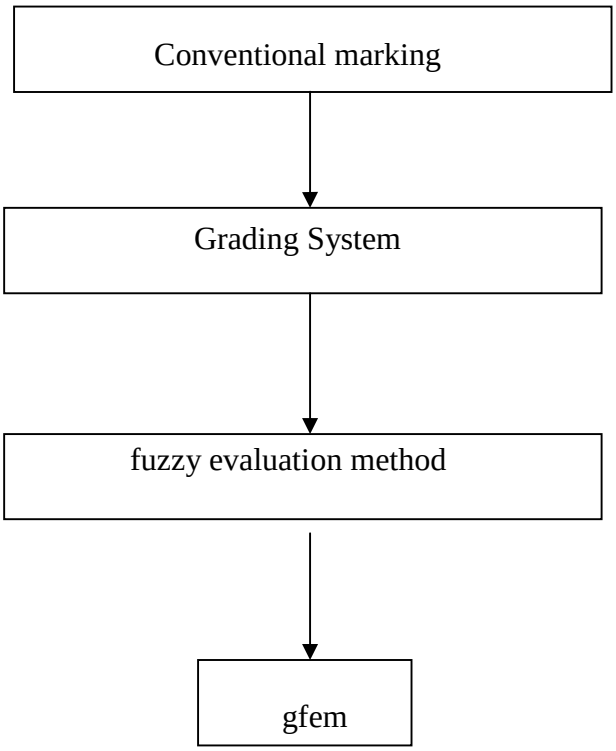
where, $_A : X \to [0, 1]$ is a mapping called the membership function of the fuzzy set A and $_A(x)$ is called the degree of belongingness or membership value or degree of membership of $x \in X$ in the fuzzy set A. We write (1) in the following form:

$A = \{\ _A(x)/x : x \in X\}$.

For briefness, however, we frequently equate fuzzy sets with their membership functions i.e. as an alternative of a fuzzy set A characterized by $_A(x)$ we will frequently say fuzzy sets $_A$.

Example. Suppose X = {6,2,0,4}. A fuzzy set of X may be given by A = {0.2/6, 1/2,0.8/0,0.1/4}.

Fig:1



Definition 2.2. *Degree of similarity between two fuzzy sets F and M:* Suppose, F and M are two fuzzy sets of a set X. Then the degree of similarity between the fuzzy sets F and M is denoted by S(F, M), and is defined by

$$S(F,M) = \frac{\hat{F}.\hat{M}}{\max(\hat{F}.\hat{F}, \hat{M}.\hat{M})},$$

where, $\hat{F}$ = ($_F(x_1)$, $_F(x_2)$, $_F(x_3)$, …..), $\hat{M}$ = ($_M(x_1)$, $_M(x_2)$, $_M(x_3)$…..), are vectors, X = {x_1, x_2, x_3,…}, and " · " denotes dot product.

If F and M are not the identical, then S(F, M) ≠1. However, they are alike up to a certain degree. How much similar depends upon the value of S(F, M). In addition, if M is a concentration of F, then F is a dilation of M and conversely. There are several measures for the degree of similarity between two fuzzy sets. However, this measure using dot product is a good measure as far as dilation-concentration is concerned.

3. VECTOR VALUED MARKING METHOD

We initiate here a new method of assessment of interviewee'sanswer scripts called as "Fuzzy evaluation method" or fem which is nothing but a vector approach. For this we initiate the concept of awarding a six-dimensional vector valued mark for each question's answer. This kind of six-dimensional vector valued mark is called fuzzy mark or fum. The following definitions are needed to proceed further.

Definition 3.1. *Universal set U:* The set U = {0, 20, 40, 60, 80, 100} is called the universal set.

Definition 3.2. *Standard Fuzzy Sets:* There are n (this n corresponds to the n-point-grading scale) Standard Fuzzy Sets of U called in short by SFS. For the present work, there are five numbers of SFS; and these are the following five fuzzy linguistic hedges:

E (excellent) = {0/0, 0/20, 0.8/40, 0.9/60, 1/80, 1/100}
= {0,0,0.8,0.9, 1, 1},

V (very good) = {0/0, 0/20, 0.8/40, 0.9/60, 0.9/80, 0.8/100}
= {0, 0, 0.8, 0.9, 0.9, 0.8},

G (good) = {0, 0.1, 0.8, 0.9, 0.4, 0.2},

S (satisfactory) = {0.4, 0.4, 0.9, 0.6, 0.2, 0},

U (unsatisfactory) = {1, 1,0.4, 0.2, 0, 0}.

(The membership values of the above SFS will be standardized by the company/organization concerned; for the presentation of this work, we just recommend those as above).

Thus we have the subsequent standard values:

E . E = 3.45, V . V = 2.90, G. G = 1.66, S. S = 1.53, and U . U = 2.20.

Definition 3.3.*Fuzzy mark or fum*: A fuzzy set of the universal set U whose graph is any one of the following type. First they may be decreasingor they may be increasingor at first increasing and then decreasing. They are represented as fum. In the present work, we shall express the fums by membership values only for the sake of simplicity.

The examples of fum representation is as follows:

(1) {0, 0, 0.6, 0.8, 0.9, 0.7},
(2) {1, 1,0.2,0,0,0},
(3) {0,0,0.2,0.4,0.4,0.6}.

A fuzzy mark (fum) like {0, 0.1, 0.2, 0.4, 0.4, 0.6} to an answer to a question shows to the respondent the degrees of evaluator's fulfillment for that answer in 0%, 20%, 40%, 60%, 80%, and 100% respectively. To avoid any human biasness of the assessor with any SFS, the evaluator at first awards rum at freedom to express his satisfaction at different levels (for example, 0%, 20%, 40%, 60%, 80% and 100%), instead of directly awarding SFS titles like Excellent, Good, etc.

Definition 3.4.*Fuzzy grade sheet:* A fuzzy grade sheet is a matrix type structure containing eight columns and i number of rows, where i is the total number of questions. A sample of a fuzzy grade sheet is shown in Table 1. At the bottom there is

Table 1

Interview Question No.	**Fuzzy Mark for interviewee**						**Grade**
Q1							
Q2							
Q3							
. . . . . .							
						Total Mark =	

a box which informs the total score. The first column exposes the serial numbers of the questions; in any row, the columns from the second to the seventh shows the fuzzy mark awarded to the answer to the corresponding question in the first column. The last i.e. eighth column shows the grade awarded to each question. The box at the bottom shows the total mark awarded to the paper. Each script will contain a fuzzy grade sheet printed on its cover page.

Definition 3.5.*Letter-grade and mid-grade-point*: "A", "B", "C', "D", "E" are called letter grades (for the present work we have been following 5-point grading method, which can be generalized if desired) which correspond to E, V, G, S, U respectively of the Standard Fuzzy Sets(SFS).

Mid-grade-points are standardized as follows: Mid-grade-point of A = 95 is denoted by P(A), B = 80 by P(B), C = 60 by P(C), D = 40, E = 15 by P(E).

Actually these are mid points of the corresponding intervals as laid down below: and can be standardized or Suitably adopted by the concerned company/organization:

$30 \leq \overline{D}$
$50 \leq C$
$70 \leq B$
$90 \leq \underline{A}$

Definition 3.6.*The procedure fem*: Consider an answer script of a candidate in an interview. Suppose an evaluator or interviewer is to evaluate the ith question i.e.Q.i of his script. The following process is to be assumed:

(1) The assessor awards a fum to Q.i by his best possible judgment and fill up the cells of the ith row for the first seven columns. declare, this fum is F_i.

(2) Now compute the following degrees of similarities:

S (E, F_i), S(V, F_i) S(G, F_i), S (S, F_i) and S (U, F_i),

where E, V, G .... are SFS.

(3) Find the maximum of the above five values. With no loss of generality, for the sake of an instance, we suppose that S (V, F_i) is maximum.

(4) Therefore, award grade "B" to Q.i, because grade "B" corresponds to V (very good) of the SFS (Definition 3.2 above). [Here V comes from S (V, F_i)].

(5) Repeat the same for the answer of each attempted question in the candidates answer script.

(6) Now calculate the total score using the following formula:

$$\text{Total score} = \frac{1}{100}\Sigma\,[T(Q.i) \times P(g_i)],$$

where, T (Q.i) is the mark allotted to Q.i in the Question paper, and g_i the grade awarded to Q. i by the evaluator.

(7) Put this total score in the suitable box at the bottom of the fuzzy grade sheet.

In case, in (4) above, maximum value occurs for two degrees, say for S (V, F_i) and S(G, F_i), the better of V and G is to be considered (for example, better of V and G is V).

3.1.An example showing how fem works

Let us illustrate fem by an example. Consider a candidate's answer script to a paper of 100 marks. In total there were four questions to be answered:

TOTAL MARKS = 100

Q.1 carries 20 marks

Q.2 carries 40 marks

Q.3 carries 10 marks

Q.4 carries 30 marks

Suppose an interviewer awards fums as shown in the following Table 2 in the fuzzy grade sheet attached with the answer script.

Calculations:

For Q.1:

$S(E, F_1) = 140/345$, $S(V, F_1) = 138/290$,
$S(G, F_1) = 128/166$, $S(S, F_1) = 106/153$,
$S(U, F_1) = 40/220$.
Max. = $S(G, F_1)$. Grade awarded = C.

For Q.2:

$S(E, F_2) = 216/345$, $S(V, F_2) = 188/290$,
$S(G, F_2) = 88/180$, $S(S, F_2) = 40/180$,
$S(U, F_2) = 8/220$.
Max. = $S(V, F_2)$. Grade awarded = B.

For Q.3:

$S(E, F_3) = 106/345$, $S(V, F_3) = 104/290$,
$S(G, F_3) = 96/166$, $S(S, F_3) = 84/153$,
$S(U, F_2) = 48/220$.
Max. = $S(G, F_3)$. Grade awarded = C.

Table 2

Interview Question No.	**Fuzzy mark for interviewee**						**Grade**
Q. 1	0	0	0.6	0.8	0.2	0	C
Q. 2	0	0	0	0.4	0.8	1	B
Q. 3	0	0.2	0.4	0.6	0.2	0	C
Q. 4	0.8	0.5	0.1	0	0	0	E
					Total mark = 55		

For Q.4:

$S(E, F_4) = 8/345$, $S(V, F_4) = 8/290$,
$S(G, F_4) = 13/166$, $S(S, F_4) = 61/153$,
$S(U, F_4) = 134/220$.
Max. = $S(U, F_4)$. Grade awarded = E.

Total score $= \frac{1}{100}\Sigma\,[T(Q.i) \times P(g_i)]$,

$= \frac{1}{100}[20 \times 60 + 40 \times 80 + 10 \times 60 + 30 \times 15]$

$= 54.5$

≈ 55

4.MATRIX VALUED MARKING METHOD

It is nothing but a generalization of fuzzy evaluation method. Here the concept of matrix valued marking method is employed. In fuzzy evaluation method, each attempted question's answer is to be awarded a fum which is nothing but a six-dimensional row vector. In contrary to fum, a generalized fuzzy mark or gfum is awarded which is a 4 × 6 type matrix. Gfum is described below:

Definition 4.1. *Generalized fuzzy mark or gfum*: A gfum is a 4 x 6 type matrix each row of which is a fum. For an answer to a question,

(i) the first row of the gfum is fum for accuracy of information,
(ii) the second row is a fum for adequate coverage,
(iii) the third row for conciseness, and
(iv) the fourth row for clear expression.

Thus gfum for the answer to a question will be like:

$$\begin{pmatrix} F_{i1} \\ F_{i2} \\ F_{i3} \\ F_{i4} \end{pmatrix}$$

Where F_{ij} are fums.

An Example of a gfum: Table 3 gives an example of a gfum for the answer to a question.

Table 3

0	**0**	**0**	**0.6**	**0.8**	**1**
0	**0**	**0**	**0.5**	**0.7**	**0.9**
0	**0.6**	**0.8**	**0.9**	**0.5**	**0.2**
0	**0.1**	**0.3**	**0.7**	**0.5**	**0**

Table 4

Question No.	**gfum**	**Grade**	**Mark**
Q.1	$\mathbf{F_{11}}$ $\mathbf{F_{12}}$ $\mathbf{F_{13}}$ $\mathbf{F_{14}}$	$\mathbf{g_{11}}$ $\mathbf{g_{12}}$ $\mathbf{g_{13}}$ $\mathbf{g_{14}}$	$\mathbf{m_1}$
Q.2			$\mathbf{m_2}$
… … …	… … …	… … …	… … …
		Total mark =	

Definition 4.2. Generalized fuzzy grade sheet (See Table 4). In the grade sheet of Table 4, for all j = 1, 2, 3, 4 and for all i, g_{ij} is the calculated grade by fem for the awarded fum F_{ij}, and m_i is the calculated mark to be awarded to the attempted Q_i using the formula:

$$m_i = \frac{1}{400} T(Q_i). \sum_{j=1}^{4} P(g_{ij})$$

and Total mark = $\Sigma\, m_i$.

5. CONCLUSION

The main aim of companies/organizations should be to provide the candidates in interview with the assessment reports in connection with their test/examination as sufficient as possible, may be with unavoidable mistakes as small as possible, in anexclusively pointed form as a numerical number instead of big intervalvalued form, as the latter does not satisfy the candidates. In the present work, we have initiated the idea of vector-valued evaluation of each question leading towards fem (fuzzy evaluation method), a computer based fuzzy approach, for overall evaluation of any answer script of the candidates in the interview. Some more areas of applications of fem are as follows:

(i) Aptitude testing - management, professional;
(ii) Evaluation of students answer scripts;
(iii) Career review/promotion;
(iv) Skill based certification;
(v) Experimental learning.

In addition, we have generalized this concept of fem by using matrix-valued marking, to introduce gfem.

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