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ON STEREOGRAPHIC SEMICIRCULAR EXPONENTIATED INVERTED WEIBULL MODEL

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ABSTRACT
From the existing life testing linear distributions a good number of Circular models were developed using a variety of techniques like wrapping, inverse stereographic projections, rising sun function etc. In this Paper a new Semicircular model called ***Stereographic Semicircular Exponentiated Inverted Weibull Distribution*** is constructed using the method of inverse stereographic projection. The Probability density function, Cumulative density function, Characteristic function, Trigonometric moments and population characteristics for the new Stereographic Semicircular Exponentiated Inverted Weibull Distribution are studied.

Keywords: Keywords: Circular Distribution, Stereographic projection, Semicircular Models, Trigonometric Moments, Exponentiated Inverted Weibull distribution.

1 INTRODUCTION

A substantial number of Circular models defined on unit circle were developed from the existing linear distributions, both continuous and discrete by Fisher (1993), Jammalamadaka and Sengupta (2001), Dattatreya Rao et al (2007). Mardia and Jupp (2000) gave expressions for population characteristics such as variance, standard deviation, skewness, kurtosis etc. for a circular distribution. A new method called Stereographic Projection was employed for the construction of new circular models. Minh and Farnum (2003) used a bilinear transformations to map points on the unit circle in the complex plane into points x on the real line and for a density function $g(\alpha)$ on the interval $(-\pi, \pi)$, and demonstrated how a corresponding density function $f(x)$ on $(-\infty, \infty)$ is induced. Toshihiro Abe et al (2010) developed symmetric circular models applying the Inverse Stereographic Projection. Dattatreya Rao et al (2011) developed Cauchy type models by inducing Stereographic Projection on circular Cardioid distribution. Phani et al (2012) developed circular model induced by Inverse Stereographic Projection on Extreme-Value distribution.

Taking cue from these works, an attempt is made in this paper to construct a Semicircular model called ***Stereographic Semicircular Exponentiated Inverted Weibull Distribution*** using the method of inverse stereographic projection.

This paper consists of four sections. Section 2 describes the methodology of inverse stereographic projection on a linear probability distribution. Section 3 derives the proposed

Stereographic Semicircular Exponentiated Inverted Weibull Distribution, and presents with the graphs of probability density, distribution and characteristic functions for different values of parameters. Prominent population characteristics for the new Stereographic Semicircular Exponentiated Inverted Weibull Distribution are also furnished. Section 4 summaries the findings of this study.

For this paper software MATLAB is used for all the computations and for plotting of graphs.

2 INVERSE STEREOGRAPHIC PROJECTION METHOD

New models for dealing with angular data can be constructed by applying inverse stereographic projection on linear probability distributions. Inverse stereographic projection when induced on a linear model yields an angular model.

Stereographic projection creates a one to one relationship between the points on the unit circle and those on the real line. Circular as well as linear Probability distributions can be obtained by applying stereographic projection. Inverse stereographic projection is defined by a one to one mapping given by $T(\theta) = x = u + v\tan\left(\frac{\theta}{2}\right)$, where $x \in (-\infty, \infty)$, $\theta \in [-\pi, \pi)$, $u \in$, *and* $v > 0$. If *x* is random variable defined on the interval $(-\infty, \infty)$ with probability density functions $f(x)$ and cumulative distribution as $F(x)$, then $T^{-1}(x) = \theta = 2\tan^{-1}\left\{\frac{(x-u)}{v}\right\}$ is a random point on a unit circle.

If $G(\theta)$ and $g(\theta)$ are respectively the cumulative distribution and the probability density functions of this random point θ, then $G(\theta)$ and $g(\theta)$ can be derived in terms of $F(x)$ and $f(x)$ using the following transformation

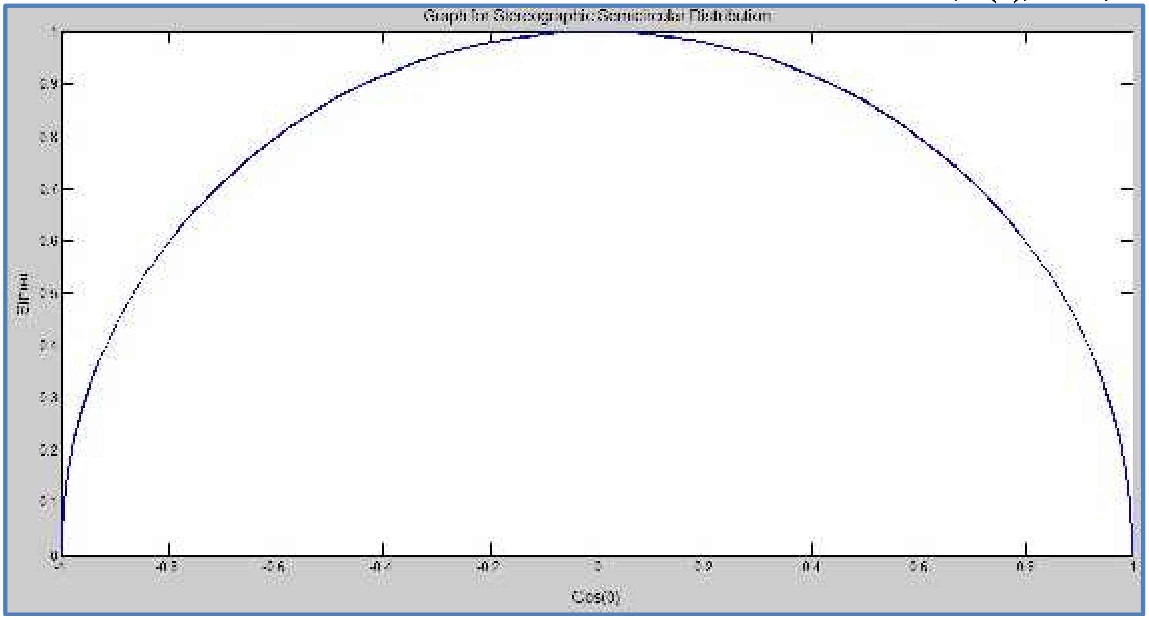
For $v > 0$

i) $$G(\theta) = F\left(u + v\tan\left(\frac{\theta}{2}\right)\right) = F\left(x(\theta)\right) \quad \ldots(1)$$

ii) $$g(\theta) = v\left(\frac{\sec^2\left(\frac{\theta}{2}\right)}{2}\right) f\left(u + v\tan\left(\frac{\theta}{2}\right)\right) \quad \ldots(2)$$

Without loss of generality *u is* considered as *zero* in this paper.

Whenever an Inverse Stereographic Projection is applied on the linear models whose probability density functions taking support on $(0,\infty)$ then the resultant distributions are Stereographic Semicircular models automatically mapped onto $(0,\pi)$. The graph placed below demonstrates this phenomenon.



3 STEREOGRAPHIC SEMICIRCULAR EXPONENTIATED INVERTED WEIBULL DISTRIBUTION

The Exponentiated Inverted Weibull distribution is a generalization to the inverted Weibull distribution through adding a new shape parameter $\lambda \in R^{+}$ by exponentiation to Inverted Weibull distribution function. A linear random variable *X* is said to follow a two parameter Exponentiated Inverted Weibull distribution, if the distribution function of *X* takes the following form

$$F(x) = \left(e^{-x^{-c}} \right)^{\lambda}$$

Where ***c*** and *λ* are shape parameters and $0 < x < \infty$ and $c > 0, \lambda > 0$

Hence the probability density function of Exponentiated Inverted Weibull distribution is

$$f(x) = \lambda . c . x^{-(c+1)} \left(e^{-x^{-c}} \right)^{\lambda} \quad \text{where } 0 < x < \infty \text{ and } c > 0, \lambda > 0$$

Here if $\lambda = 1$, this EIW distribution becomes the standard Inverted Weibull distribution and if $c = 1$ this distribution represents standard Inverted Exponential distribution.

a) Probability density function for Stereographic Semicircular Exponentiated Inverted Weibull (SSEIW) distribution:

As discussed in the previous section, by applying the inverse stereographic projection on the Exponentiated Inverted Weibull distribution, the one to one mapping of

$$T(\theta) = x = v \tan\left(\frac{\theta}{2}\right); \; where \;\; x \in (0, \infty), \theta \in [0, \pi) \text{ and } v > 0$$

will yield the Stereographic Semicircular Exponentiated Inverted Weibull (SSEIW) distribution with probability density function g (θ) and cumulative distribution function G (θ) as below:

$$1.\ g(\theta) = \frac{v}{2}\sec^{2}\left(\frac{\theta}{2}\right)\lambda . c . \left(v\tan\left(\frac{\theta}{2}\right)\right)^{-(c+1)}\left(e^{-\left(v\tan\left(\frac{\theta}{2}\right)^{-c}\right)}\right)^{\lambda} .$$

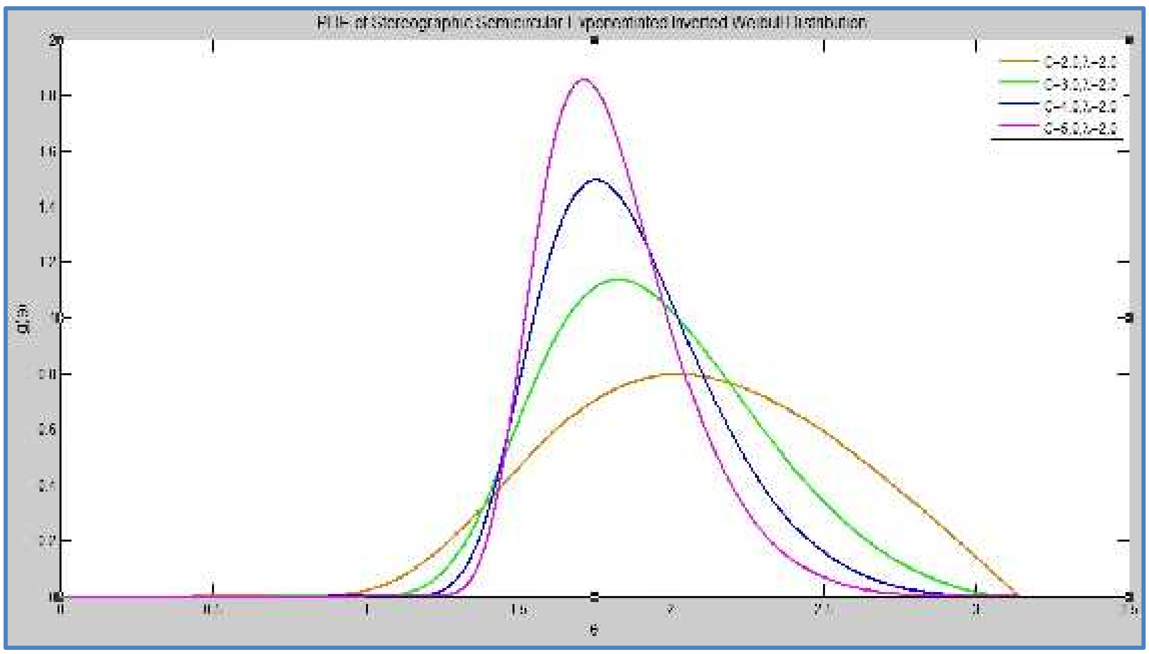
and

$$2. G(\theta) = \left(e^{-\left(v\tan\left(\frac{\theta}{2}\right)^{-c}\right)}\right)^{\lambda}$$

$$where\ \theta \in (0,\pi), c, \lambda > 0, and\ v \in \mathbb{R}^{+}$$

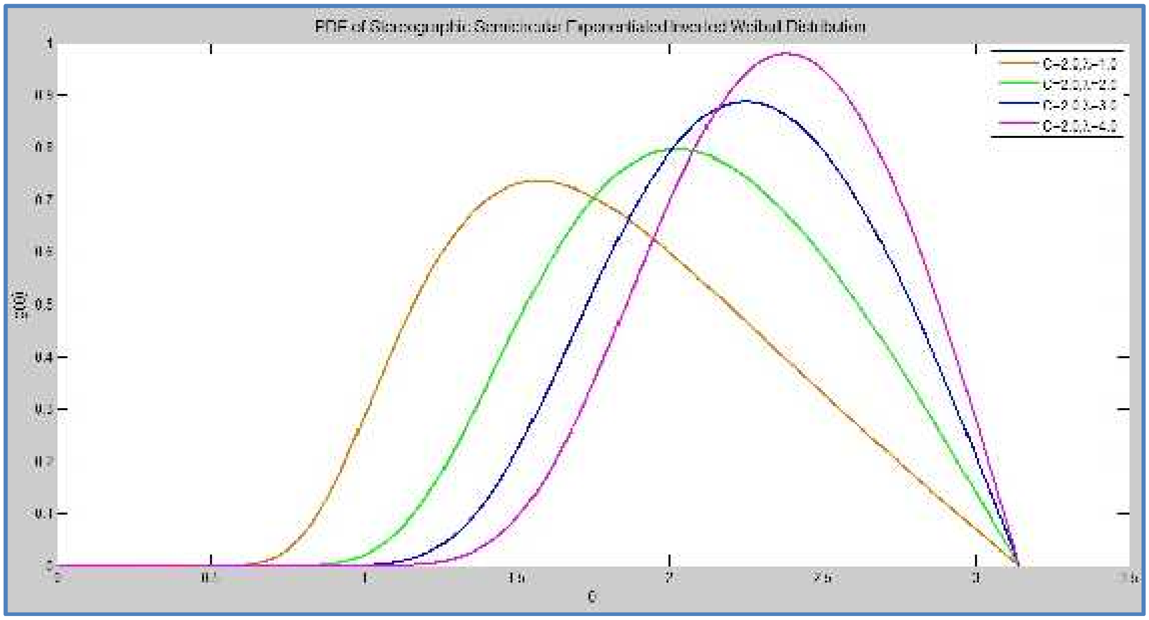
The graph depicting the linear representation of the pdf of SSEIW distribution for different values of ***c*** keeping the value for the parameter **λ** at 2.0 is as follows:

Figure 1: PDF of SSEIW distribution (Linear Representation)



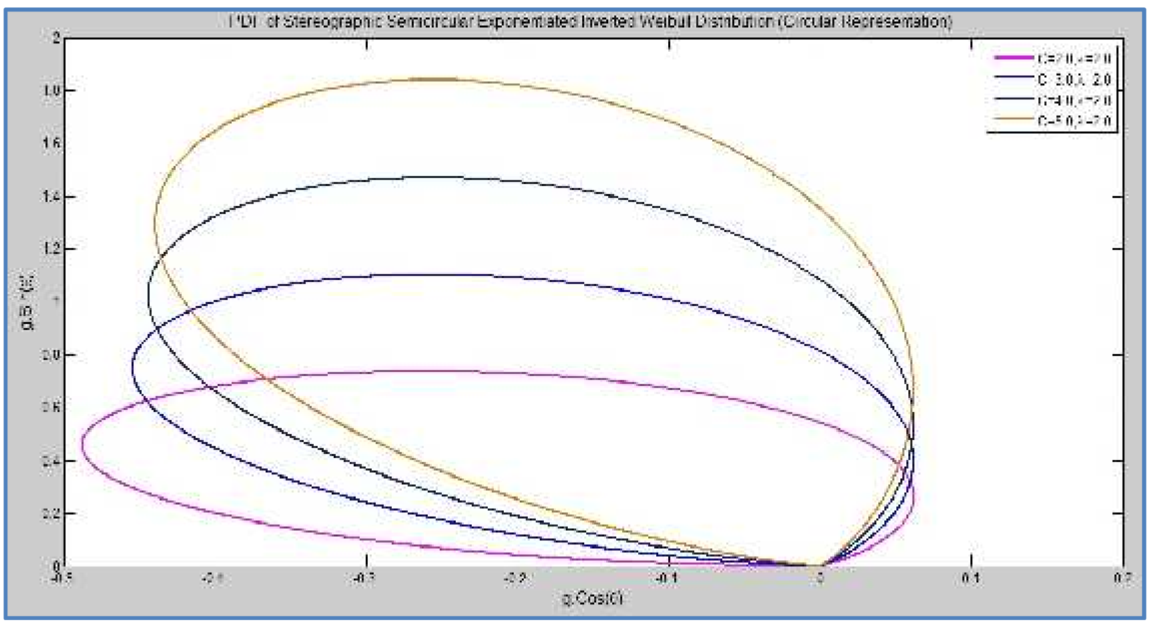
Also the linear representation of the pdf for different values of **λ** keeping the parameter ***c*** at 2.0 is obtained as below

Figure 2: PDF of SSEIW distribution (Linear Representation)



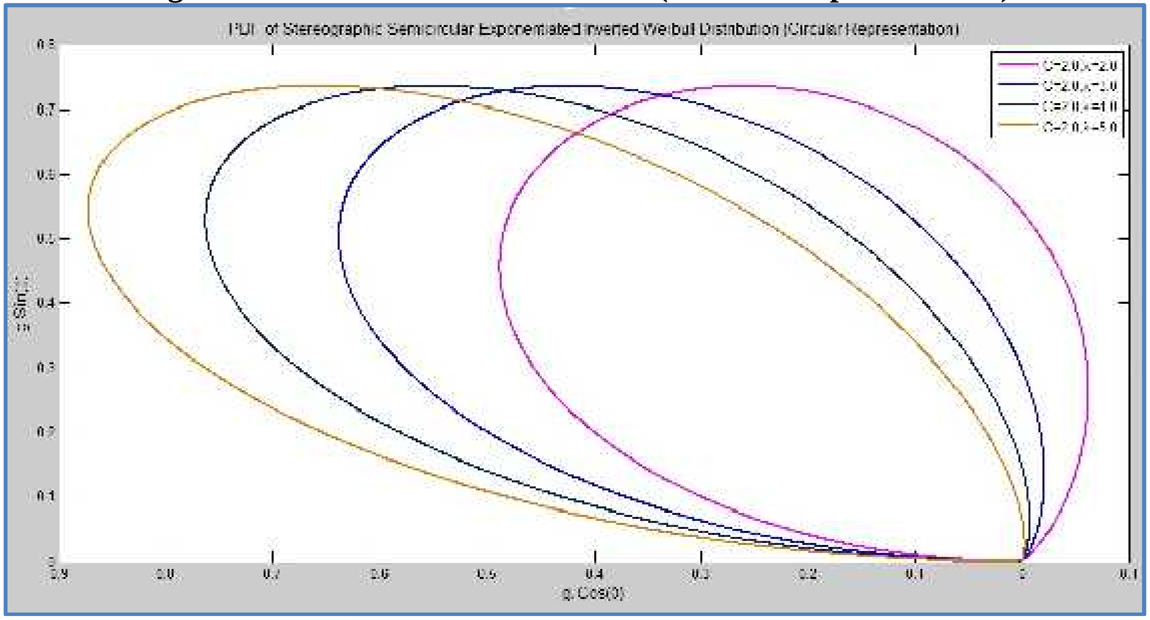
Also the graph depicting the circular representation of the pdf of SSEIW Distribution for different values of ***c*** keeping the value for the parameter **λ** at 2.0 is shown below:

Figure 3: PDF of SSEIW distribution (Circular Representation)



The circular representation for the pdf for different values of λ keeping the parameter ***c*** at 2.0 is obtained as below:

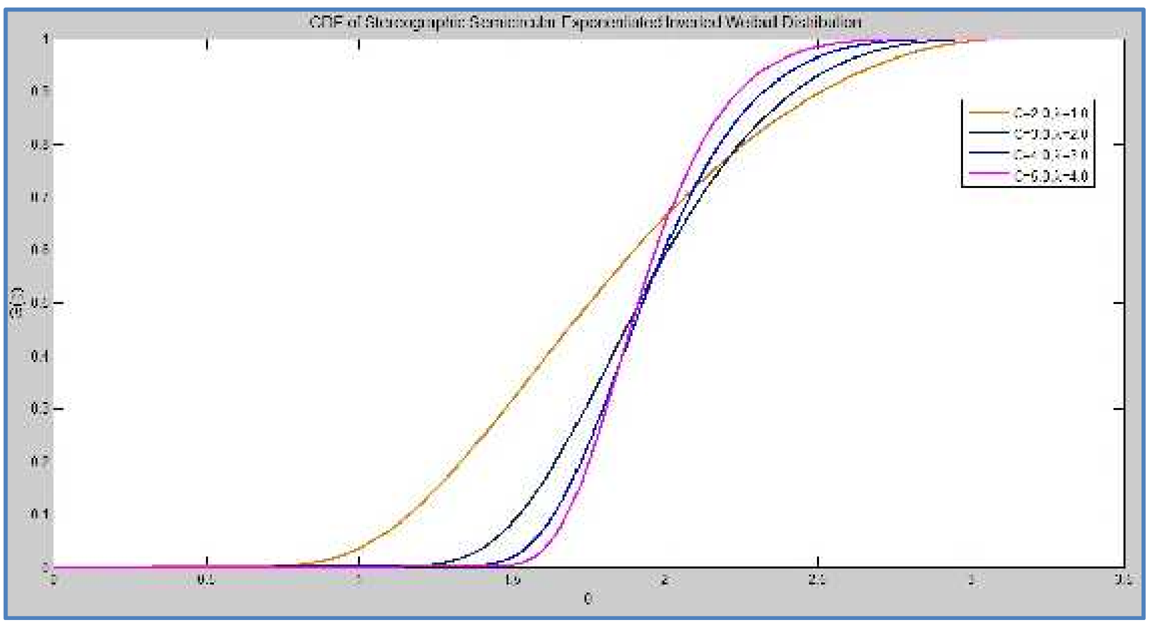
Figure 4: PDF of SSEIW distribution (Circular Representation)



b) Cumulative Distribution Function (CDF) for SSEIW distribution:

The graph for the CDF, $G(\theta)$ for SSEIW is obtained as below:

Figure 5: CDF of SSEIW distribution



c) Characteristic function for SSEIW distribution:

The characteristic function of a Stereographic Semicircular model

The characteristic function of a Semicircular model with the probability density function $g(^{-})$ is defined as

$$\ddot{}_{p} = \int_{0}^{®} e^{ip¯} g(¯) d¯ \ , p \in$$

If the cumulative distribution function and probability density function for a Stereographic Semicircular model are $G(¯)$ and $g(¯)$ respectively and if $F(x)$ and $f(x)$ are the corresponding cumulative distribution function and probability density function of the linear model, then for the Stereographic Semicircular model the characteristic function can obtained as

$$\ddot{}_{p} = \int_{0}^{®} e^{ip¯} g(¯) d¯ \ , p \in$$

Without loss of generality, considering v =1, the characteristic function for the Stereographic Semicircular Exponentiated Inverted Weibull distribution can be obtained from the following equation.

$$\ddot{}_{p} = \int_{0}^{®} e^{ip¯} \ \frac{1}{2}\sec^{2}\left(\frac{¯}{2}\right) a.c.\left(\tan\left(\frac{¯}{2}\right)\right)^{-(c+1)} \left(e^{-\left(\tan\left(\frac{¯}{2}\right)^{-c}\right)}\right)^{a} d¯ \qquad \ldots (3)$$

Taking $u = \left(\tan\left(\frac{¯}{2}\right)\right)^{-(c)}$ and transforming (3) we get

$$\ddot{}_{p} = \int_{0}^{\infty} e^{ip(2\tan^{-1}(u^{(1/c)})_{a}} \left(e^{-u}\right)^{a} du \qquad \ldots (4)$$

Again taking k= uλ and making necessary transformations for (4) the characteristic function for the Stereographic Semicircular Exponentiated Inverted Weibull distribution can be obtained as below:

$$\ddot{}_{p} = \int_{0}^{\infty} e^{2ip\tan^{-1}\left(\left(\frac{k}{a}\right)^{(1/c)}\right)} e^{-k} dk \qquad \ldots (5)$$

It can be can see that the integral obtained above cannot be evaluated analytically in its general form, for evaluating the characteristic function to obtain the trigonometric moments, the n – point Gauss – Laguerre quadrature formula for numerical integration as given in Rao et al, (1975) is applied on equation (5). The real and imaginary parts $\sim_{p}$ and $_{p}$ respectively are obtained from the characteristic function of the SSEIW distribution. The following are the graphs for the characteristic function of the SSEIW distribution showing the real part and imaginary part separately for different values of c and λ.

Figure 6: Characteristic Function of SSEIW distribution at c=2, λ=2

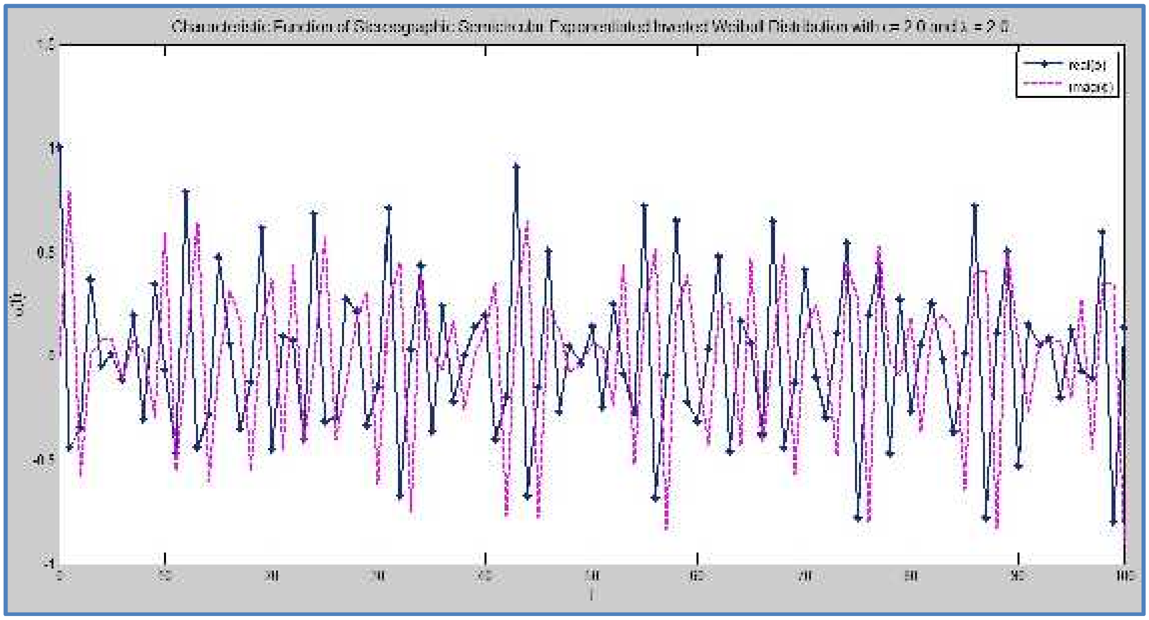
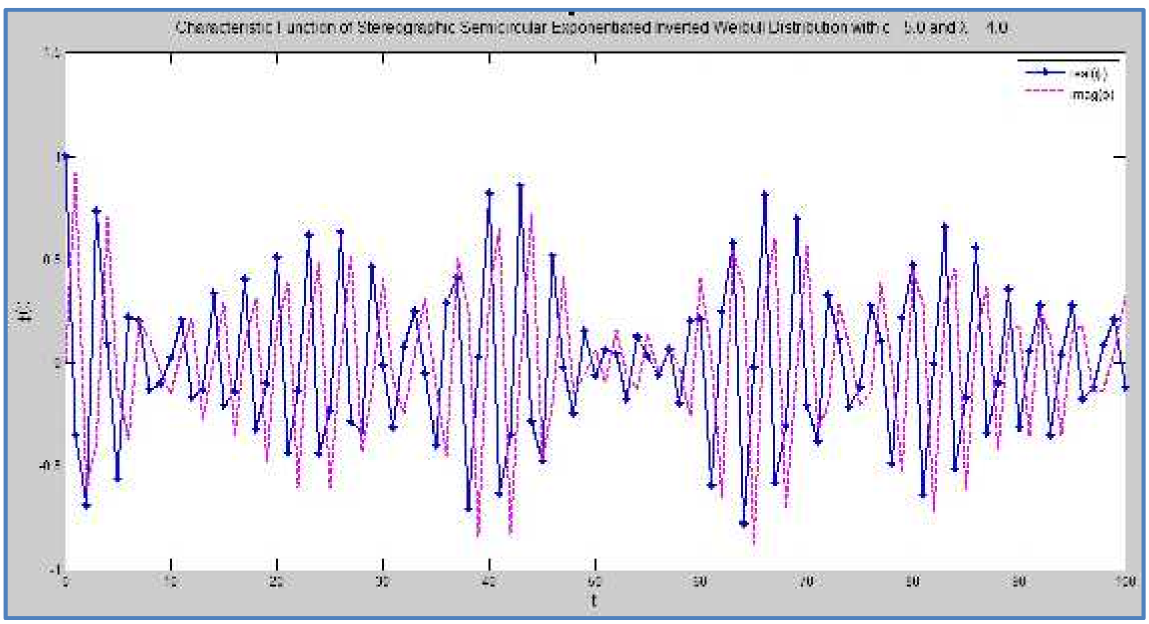


Figure 7: Characteristic Function of SSEIW distribution at c=5, λ=4



d) Population Characteristics:

Mardia and Jupp (2000) had derived expressions for mean direction μ_o resultant length ρ_1, Circular variance V_0, Central Trigonometric Moments α_p^*, β_p^*, Skewness γ_1^o and Kurtosis γ_2^o from the values of the characteristic function φ_p at p=1 and 2.

The Population Characteristics for the Stereographic Semicircular Exponentiated Inverted Weibull Distribution for different values of the parameters *c* and λ are computed and tabulated here under.

Table 1: Characteristics of Stereographic Semicircular Exponentiated Inverted Weibull Distribution at λ = 3.0

At different values of c	c=2.0	c=2.5	c=3.0	c=3.5	c=4.0
Trigonometric Moments					
α_1	-0.5725	-0.5034	-0.4462	-0.3989	-0.3596
α_2	-0.1600	-0.3319	-0.4636	-0.5643	-0.6417
β_1	0.7276	0.7959	0.8432	0.8767	0.9010
β_2	-0.7089	-0.7066	-0.6816	-0.6464	-0.6078
Resultant Length					
ρ_1	0.9258	0.9418	0.9540	0.9632	0.9701
ρ_2	0.7268	0.7807	0.8243	0.8580	0.8839
Mean Direction					
μ_0	2.2375	2.1347	2.0575	1.9978	1.9505
Variance					
V_0	0.0742	0.0582	0.0460	0.0368	0.0299
Circular Standard Deviation					
σ_0	0.3926	0.3464	0.3069	0.2738	0.2463
	0.7989	0.7037	0.6216	0.5534	0.4969
Circular Trigonometric Moments					
α_1^*	0.9258	0.9418	0.9540	0.9632	0.9701
α_2^*	0.7267	0.7807	0.8243	0.8580	0.8839
β_1^*	0.0000	0.0000	0.0000	0.0000	0.0000
β_2^*	0.0112	0.0030	0.0001	-0.0008	-0.0010
Skewness					
Υ_1^0	0.5558	0.2158	0.0130	-0.1158	-0.2023
Kurtosis					
Υ_2^0	-1.4598	-1.7428	-1.9024	-2.0029	-2.0706

Table 2: Characteristics of Stereographic Semicircular Exponentiated Inverted Weibull Distribution at c = 3.0

At different values of c	λ=2.0	λ=2.5	λ=3.0	λ=3.5	λ=4.0
Trigonometric Moments					
α_1	-0.3427	-0.4008	-0.4462	-0.4831	-0.5138
α_2	-0.5961	-0.5266	-0.4636	-0.4068	-0.3555
β_1	0.8847	0.8631	0.8432	0.8251	0.8086
β_2	-0.5411	-0.6225	-0.6816	-0.7260	-0.7601
Resultant Length					
ρ_1	0.9488	0.9516	0.9540	0.9561	0.9580
ρ_2	0.8051	0.8153	0.8243	0.8322	0.8392
Mean Direction					
μ_0	1.9403	2.0055	2.0575	2.1004	2.1369
Variance					
V_0	0.0512	0.0484	0.0460	0.0439	0.0420
Circular Standard Deviation					
σ_0	0.3243	0.3151	0.3069	0.2995	0.2928
	0.6585	0.6390	0.6216	0.6061	0.5922
Circular Trigonometric Moments					
α_1^*	0.9488	0.9516	0.9540	0.9561	0.9580
α_2^*	0.8051	0.8153	0.8243	0.8322	0.8392
β_1^*	0.0000	0.0000	0.0000	0.0000	0.0000
β_2^*	-0.0016	-0.0006	0.0001	0.0006	0.0010
Skewness					
Υ_1^0	-0.1406	-0.0555	0.0130	0.0699	0.1182
Kurtosis					
Υ_2^0	-1.9810	-1.9387	-1.9024	-1.8706	-1.8423

4 CONCLUSION

The Stereographic Semicircular Exponentiated Inverted Weibull distribution becomes Stereographic Semicircular Inverted Weibull distribution when **λ** = 1 and when c =1, Stereographic Semicircular Exponentiated Inverted Weibull distribution becomes Stereographic Semicircular Exponentiated Inverted Exponential distribution.

It can be seen from the population characteristics for the Stereographic Semicircular Exponentiated Inverted Weibull distribution tabulated in the previous section, that with increasing value of shape parameter ***c*,** keeping other shape parameters **λ = 3.0,** the circular variance gradually decreased, the distribution shifted from positively skewed to negatively skewed one and remained platykurtic. Also with the increasing value of the scale parameter λ at ***c*** = **3.0,** the Circular variance gradually decreased, the distribution shifted from negatively skewed to positively skewed one and remained platykurtic.

REFERENCES

1. Dattatreya Rao, A.V., Ramabhadra Sarma, I., Girija S.V.S., (2007). On wrapped version of some life testing models. Communication in Statistics - Theory and Methods, 36, 2027-2035
2. Dattatreya Rao A.V, Girija S.V.S, Phani. Y. (2011) “Differential Approach to Cardioid Distribution”, Computer Engineering and Intelligent Systems, Vol/Issue: 2(8). Pp. 1-6, 2011.
3. Fisher, N. I. (1993). Statistical Analysis of Circular Data. Cambridge University Press, Cambridge.
4. Flaih.A, H. Elsalloukh, E. Mendi and M. Milanova (2012), The Exponentiated Inverted Weibull Distribution, Appl. Math. Inf. Sci. 6, No. 2, 167-171
5. Girija, S.V.S., 2010. Construction of New Circular Models. VDM - VERLAG, Germany.
6. Jammalamadaka S. Rao, Sen Gupta, A., (2001). Topics in Circular Statistics, World Scientific Press, Singapore.
7. Mardia, K.V. and Jupp, P.E. (2000), Directional Statistics, John Wiley, Chichester.
8. Minh, Do Le and Farnum, Nicholas R. (2003), Using Bilinear Transformations to Induce Probability Distributions, Communication in Statistics – Theory and Methods, 32, 1, pp. 1 – 9.
9. Phani. Y., Girija S.V.S and Dattatreya Rao A.V. (2012) Circular Model Induced by Inverse Stereographic Projection On Extreme-Value Distribution, IRACST – Engineering Science and Technology: An International Journal (ESTIJ), Vol.2, No. 5, 2250-3498
10. Ramabhadra Sarma, I., Dattatreya Rao, A.V. and Girija S.V.S., (2009). On Characteristic Functions of Wrapped Half Logistic and Binormal Distributions, International Journal of Statistics and Systems, Vol 4(1), pp. 33–45.
11. Rao, C.R. & Mitra, S.K. (1975), Formulae and Tables for Statistical Work, Statistical Publishing Society.
12. Toshihiro Abe, Kunio Shimizu, Arthur Pewsey (2010). “Symmetric Unimodal Models for Directional Data Motivated by Inverse Stereographic Projection”, J. Japan Statist. Soc., Vol/Issue: 40(1). Pp 45-61.